

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY- GURAJADA VIZIANAGARAM
II B. Tech I Semester Regular Examinations, November – 2024
Random Variables and Stochastic Process
(ECE)

Time: 3 hours

Max. Marks: 70

Question paper consists of Part A, Part B.
Part A is compulsory, Answer all questions.
In Part B, Answer any one question from each unit.

PART-A**(20 Marks)**

- | | | | |
|---|----|---|-----|
| 1 | a) | What are the Conditions for a Function to be a Random Variable? | [2] |
| | b) | What is the application of Exponential Random Variable? | [2] |
| | c) | Define Skew? | [2] |
| | d) | What is Transformation Random Variable? | [2] |
| | e) | Define Marginal Density Function? | [2] |
| | f) | What is Central Limit Theorem? | [2] |
| | g) | Define Deterministic Random Processes | [2] |
| | h) | Define Strict- Sense Stationary. | [2] |
| | i) | What is Spectrum? | [2] |
| | j) | What is the System Response of Convolution for Linear Systems? | [2] |

PART-B**(50 Marks)****Unit-1**

- | | | | |
|---|-----|---|-----|
| 2 | a) | Define conditional probability distribution function and write the properties | [5] |
| | b) | A production line manufactures 5-gal gasoline cans to a volume tolerance of 5%. The probability of any one can being out of tolerance is 0.03. if four cans are selected at random: | [5] |
| | i. | What is the probability they are all out of tolerance? | |
| | ii. | What is the probability of exactly two being out? | |

(OR)

- | | | | |
|---|----|--|-----|
| 3 | a) | Define Random variable? List out the properties of Distribution Function | [5] |
| | b) | Suppose height to the bottom of clouds is a Gaussian random variable for which $\mu = 4000\text{m}$ and $\sigma = 1000\text{m}$. A person bets that cloud height tomorrow will fall in the set $A = \{1000\text{m} < X \leq 3000\text{m}\}$ while a second person bets that height will be satisfied by $B = \{2000\text{m} < X \leq 4200\text{m}\}$. A third person bets they are both correct. Find the probability that each person will win the bet. | [5] |

Unit-2

- | | | | |
|---|----|--|-----|
| 4 | a) | A random variable X is uniformly distributed on the interval $(-\pi, \pi)$. X is transformed to the new random variable $Y = T(x) = a \tan(X)$, where $a > 0$. Find the probability density function of | [5] |
| | b) | State and prove Chebychev's inequality | [5] |
- (OR)
- | | | | |
|---|----|--|-----|
| 5 | a) | Discuss in detail about Functions that gives Moments? | [5] |
| | b) | Find the mean, variance from moment generation function of uniform distribution? | [5] |

Unit-3

- 6 a) Define Marginal density function? Find the Marginal density functions of below joint density function. [5]

$$f_{XY} = \frac{1}{12} u(x)u(y)e^{-x/3}e^{-y/4}$$

- b) Two statistically independent random variable X and Y have mean values $\bar{X}=E[X]=2$ and $E[Y]=4$. They have second moments $\bar{X}^2=E[X^2]=8$ and $E[Y^2]=25$. Find i) the mean value ii) the second moment iii) the variance of the random variable $W=3X-Y$. [5]

(OR)

- 7 a) Joint Sample Space has three elements (1, 1), (2, 2), and (3, 3) with probabilities 0.4, 0.3, 0.3 respectively then draw the Joint Distribution Function diagram [4]
- b) Gaussian random variable X_1 and X_2 for which $\bar{X}_1=2, \sigma_{X_1}^2=9, \bar{X}_2=-1, \sigma_{X_2}^2=4$ and $C_{X_1X_2}=-3$ are transformed to new random variable Y_1 and Y_2 according to $Y_1=-X_1+X_2, Y_2=-2X_1-3X_2$. Find i) $\sigma_{Y_1}^2$ ii) $\sigma_{Y_2}^2$ iii) $C_{Y_1Y_2}$. [6]

Unit-4

- 8 a) Write the properties of Cross correlation Function of Random Process [5]
- b) Given that the autocorrelation function for a stationary Ergodic process with no period components is [5]

$$R_{xx}(\tau) = 25 + \frac{4}{1 + 6\tau^2}$$

Find the mean and variance of process X(t)?

(OR)

- 9 a) Write the properties of Autocorrelation Function of Random Process [5]
- b) Give the random process by $X(t)=A \cos(w_0t) + B \sin(w_0t)$ Where w_0 is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the same variance, show that X(t) is wide sense stationary but not strictly stationary [5]

Unit-5

- 10 a) The autocorrelation function of a random process X(t) [5]

$$R_{xx}(\tau) = 3 + 2 \exp(-4\tau^2)$$

- i. Find the power spectrum of X(t)
- ii. What is the average power in X(t)?
- b) Assume X(t) is a wide sense stationary process with non zero mean value. show that [5]

$$s_{xx}(\omega) = 2\pi \bar{X}^2 \delta(\omega) + \int_{-\infty}^{\infty} C_{xx}(\tau) e^{-j\omega\tau} d\tau$$

Where $C_{xx}(\tau)$ is the auto covariance function of X(t).

(OR)

- 11 a) Derive the relationship between cross-power spectrum and cross-correlation [10]
